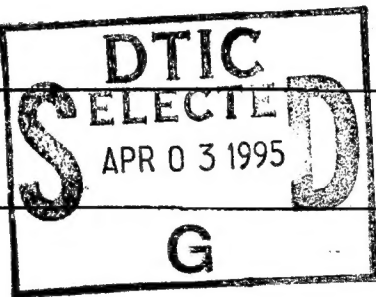


REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188	
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.				
1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED Period of Performance INITIALS OF FINAL		
4. TITLE AND SUBTITLE Inverse Scattering Problems for Acoustic and Electromagnetic Waves (U)		5. FUNDING NUMBERS F49620- 92-J-0048 (Grant/Contract Number)		
6. AUTHOR(S) David L. Colton & Peter B. Monk				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) University of Delaware Newark, DE 19716		8. PERFORMING ORGANIZATION REPORT NUMBER AFOSR-TR-95-0179		
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) AFOSR/NM 110 DUNCAN AVE SUITE B115 BOLLING AFB DC 20332-0001		10. SPONSORING/MONITORING AGENCY REPORT NUMBER F49620-92-J-0048		
11. SUPPLEMENTARY NOTES				
12a. DISTRIBUTION/AVAILABILITY STATEMENT APPROVED FOR PUBLIC RELEASE: DISTRIBUTION UNLIMITED				
12b. DISTRIBUTION CODE UL				
13. ABSTRACT (Maximum 200 words) This research project was concerned with the inverse scattering problem for time harmonic electromagnetic waves. The main accomplishments were 1) the introduction of modified far field operators to eliminate unwanted eigenvalues in the dual space method for solving the inverse problem, 2) the numerical analysis of a combined finite element-boundary element method for solving the direct scattering problem, 3) an analysis of the spectral properties of the far field operator and the physical meaning of the distribution of eigenvalues, and 4) the adaptation of the dual space method to the problem of the detection and monitoring of leukemia.				
14. SUBJECT TERMS		15. NUMBER OF PAGES 7		16. PRICE CODE
17. SECURITY CLASSIFICATION OF REPORT UNCLASSIFIED	18. SECURITY CLASSIFICATION OF THIS PAGE UNCLASSIFIED	19. SECURITY CLASSIFICATION OF ABSTRACT UNCLASSIFIED	20. LIMITATION OF ABSTRACT SAR	

Standard Form 298 (Rev. 2-89)  
Prescribed by ANSI Std. Z39-18  
298-102

19950328 096

THIS QUARTER ENCLOSED L

## Final Report

AFOSR Grant F49620-92-J-0048

November 1, 1991 – October 31, 1994

Summary: This research project was concerned with the inverse scattering problem for time harmonic electromagnetic waves. The main accomplishments were 1) The introduction of modified far field operators to eliminate unwanted eigenvalues in the dual space method for solving the inverse problem, 2) The numerical analysis of a combined finite element-boundary element method for solving the direct scattering problem, 3) An analysis of the spectral properties of the far field operator and the physical meaning of the distribution of eigenvalues and 4) the adaptation of the dual space method to the problem of the detection and monitoring of leukemia.

### Professional Personnel

Dr. David Colton      Professor of Mathematics  
University of Delaware  
Co-Principal Investigator

Dr. Peter Monk      Professor of Mathematics  
University of Delaware  
Co-Principal Investigator

Accession For	
NTIS	CRA&I <input checked="" type="checkbox"/>
DTIC	TAB <input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification _____	
By _____	
Distribution / _____	
Availability Codes	
Dist	Avail and/or Special
A-1	

## Report on Research

The research of AFOSR Grant F49620-92-J-0048 was concerned with the development of new methods to solve the inverse scattering problem for time harmonic electromagnetic waves. The main problem under consideration was the determination of the permittivity and conductivity of an isotropic inhomogeneous medium from a knowledge of the electric far field data of the scattered wave. Our approach to this problem has been through the use of Herglotz wave functions and finite element methods which yield what we call the *dual space method* to solve the inverse problem. The main advantage of this method is that by means of an averaging procedure the number of unknowns in the nonlinear optimization scheme for determining the index of refraction is reduced by roughly 75%.

Prior to this research project, a major problem with the numerical implementation of the dual space method was that it failed at certain values of the wave number called transmission eigenvalues. Initial attempts to resolve this problem did so by introducing a new (but more manageable) eigenvalue problem. A complete solution to this problem was provided in [4], [12] and [14] where the far field operator was modified in such a way as to avoid any problems with undesirable eigenvalues. This approach was extended to near field data and an absorbing host medium in [15]. With these modifications, the dual space method is now applicable for any positive value of the wave number for both absorbing and non-absorbing host media.

A second problem with the numerical implementation of the dual space method was that it was originally formulated in terms of a volume integral equation over an inhomogeneous region (subject to a constraint arising from the averaging procedure applied to the far field data). A closely related volume integral equation is also often used in generating synthetic far field data to test the efficiency of various approaches for solving the inverse problem. In our opinion, a more efficient method for implementing the dual space method as well as for solving scattering problems in an inhomogeneous medium is through the use of coupled finite element-boundary element methods or coupled finite-element-spectral methods. These approaches were analyzed in [17] and [18] and incorporated into the numerical algorithm which implements the dual space method.

The canonical model for inverse scattering theory uses the far field pattern as data. Hence, for both mathematical and practical reasons, it is of interest to determine what information about the scattering object can be obtained from specialized knowledge of either the far field pattern or the far field operator (the integral operator defined over the unit sphere with far field pattern as kernel). In [16] it was shown that the far field pattern uniquely determines the scattering object and in [8] and [9] the spectral theory of the far field operator for a conducting medium was investigated. In these papers it was shown that the eigenvalues of the far field operator exist and lie inside a disk in the complex plane. It was further shown that the radius of this disk gives information on the magnitude of the permittivity and conductivity of the scattering object.

In the past year, particular concern has been given to the adaptation of the dual space method to the problem of the detection or monitoring of leukemia. As suggested to us by Dr. Richard Albanese of the Mathematical Products Division of the USAF Armstrong Laboratory, increases in the cellular population of bone marrow are reflected as increases or decreases in the permittivity and conductivity respectively. Hence, this change can potentially be detected by electromagnetic imaging. Using a two dimensional model (e.g. applicable to the monitoring of bone marrow in the upper leg) and synthetic data, we have shown that this approach is potentially viable ([13], [15]). Research in this direction is continuing.

A survey of our work on inverse scattering theory up to October 1992 under Air Force support has appeared as Volume 93 of the Springer-Verlag Series on Applied Mathematics ([7]).

#### Publications

1. G. Cohen and P. Monk, Gauss point mass lumping schemes in electromagnetism, *Compel 13, Supplement A* (1994), 293-298.
2. D. Colton, The inverse scattering problem for electromagnetic waves, in *Inverse Problems in Scattering and Imaging*, M. Bertero and E. Pike, editors, Adam Hilger, Bristol, 1992, 59-70.

3. D. Colton, Target signatures for Maxwell's equations, in *Inverse Problems in Mathematical Physics*, L. Päiväranta and E. Somersalo, editors, Springer-Verlag Lecture Notes in Physics, Vol. 422, Berlin, 1993, 22-28.
4. D. Colton and P. Hähner, Modified far field operators in inverse scattering theory, *SIAM J. Math. Anal.* 24 (1993), 365-389.
5. D. Colton and A. Kirsch, The use of polarization effects in electromagnetic inverse scattering problems, *Mathematical Methods in the Applied Sciences* 15 (1992), 1-10.
6. D. Colton and A. Kirsch, Target signatures for imperfectly conducting obstacles at fixed frequency, *Quart. Jour. Mech. Appl. Math* 47 (1994), 1-15.
7. D. Colton and R. Kress, *Inverse Acoustic and Electromagnetic Scattering Theory*, Springer-Verlag, Berlin, 1992.
8. D. Colton and R. Kress, Eigenvalues of the far field operator and inverse scattering theory, *SIAM J. Math. Anal.*, to appear.
9. D. Colton and R. Kress, Eigenvalues of the far field operator for the Helmholtz equation in an absorbing medium. *SIAM J. Appl. Math.*, to appear.
10. D. Colton and P. Monk, A comparison of two methods for solving the inverse scattering problem for acoustic waves in an inhomogeneous medium, *J. Comp. Appl. Math.* 42 (1992), 5-16.
11. D. Colton and P. Monk. The numerical solution of an inverse scattering problem for acoustic waves, *IMA Jour. Appl. Math.* 49 (1992), 163-184.
12. D. Colton and P. Monk, On a class of integral equations of the first kind in inverse scattering theory, *SIAM J. Appl. Math.* 53 (1993), 847-860.
13. D. Colton and P. Monk, The monitoring of human or animal bone marrow by electromagnetic waves, in *Inverse Problems: Principles and Applications in Geophysics, Technology and Medicine*, G. Anger, et. al. editors, Akademie Verlag, Berlin, 1993, 100-110.
14. D. Colton and P. Monk, A modified dual space method for solving the electromagnetic inverse scattering problem for an infinite cylinder, *Inverse Problems* 10 (1994), 87-107.
15. D. Colton and P. Monk, The detection and monitoring of leukemia using electromagnetic waves: mathematical theory, *Inverse Problems*, to appear.

16. D. Colton and L. Päiväranta, The uniqueness of a solution to an inverse scattering problem for electromagnetic waves, *Arch. Rat. Mech. Anal.* **119** (1992), 59–70.
17. A. Kirsch and P. Monk, An analysis of finite element and Nyström methods in acoustic scattering, *IMA Jour. Numerical Anal.* **14** (1994), 523–544.
18. A. Kirsch and P. Monk, A finite element/spectral method for approximating the time harmonic Maxwell system in  $\mathbb{R}^3$ , *SIAM J. Appl. Math.*, to appear.
19. Ch. G. Makridakis and P. Monk, Time-discrete finite element schemes for Maxwell's equations, *RAIRO M<sup>2</sup>AN*, to appear.
20. P. Monk, A mixed method for approximating Maxwell's equations, *SIAM J. Numer. Anal.* **28** (1991), 1610–1634.
21. P. Monk, Analysis of a finite element method for Maxwell's equations, *SIAM J. Numer. Anal.* **29** (1992), 714–729.
22. P. Monk, A comparison of three mixed methods for the time-dependent Maxwell's equations, *SIAM J. Sci. Stat. Comp.* **13** (1992), 1097–1122.
23. P. Monk, A finite element method for approximating the time harmonic Maxwell's equations, *Numerische Mathematik* **63** (1992), 243–261.
24. P. Monk, An analysis of Nédélec's method for the spatial discretization of Maxwell's equations, *J. Comp. Appl. Math.* **47** (1993), 101–121.
25. P. Monk, Nonlinear acoustic tomography, in *Nonlinear Processes in Physics*, A. S. Fokas, et.al editors, Springer-Verlag, Berlin, 1993, 334–338.
26. P. Monk, Finite element time domain methods for Maxwell's equations, in *Mathematical and Numerical Aspects of Wave Propagation*, R. Kleinman, et. al. editors, SIAM, Philadelphia, 1993, 380–389.
27. P. Monk, On the  $p$  and  $hp$  extension of Nedlec's curl conforming elements, *J. Comp. Appl. Math.*, **53** (1994), 117–137.
28. P. Monk, Superconvergence of finite element approximations to Maxwell's equations, *Numer. Math PDE* **10** (1994), 793–812.
29. P. Monk, A dispersion analysis of finite element methods on triangular grids for Maxwell equations, in *The Mathematics of Finite Elements and Applications*, J. R. Whiteman, editor, John Wiley, New York, 1994, 315–321.

30. P. Monk and A. K. Parrott, A dispersion analysis of finite element methods for Maxwell's equations, *SIAM J. Sci. Comp.* **15** (1994), 916–937.
31. P. Monk, A. K. Parrott and P. Wesson, A parallel electromagnetic scattering code, *COMPEL 13, Supplement A* (1994), 237–242.
32. P. Monk and E. Süli, A convergence analysis of Yee's scheme on non-uniform grids, *SIAM J. Numer. Anal.* **31** (1994), 393–412.
33. P. Monk and S. Y. Zhang, Multigrid computation of vector potentials, *J. Comp. Appl. Math.*, to appear.